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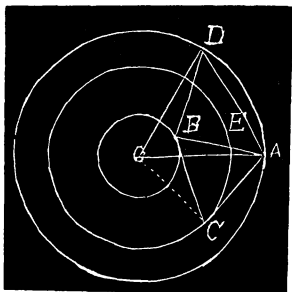
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For, in the two triangles ABD and ACO ,
 $AD=AO$, $BD=OC$, $AB=AC$.

$\therefore \triangle ABD = \triangle ACO$; $\therefore \angle DAB = \angle OAC$.

$\therefore \angle DAO = \angle BAC = 60^\circ$.

$\therefore \triangle ABC$ is an equilateral triangle having its vertices in the three concentric circumferences.

G. W. Greenwood called attention to the similarity of this problem to No. 250.

258. Proposed by B. F. FINKEL, A. M., Professor of Mathematics, Drury College, Springfield, Mo.

Prove that the tangents to an ellipse from any external point subtend equal angles at the focus, by means of the formula $\tan \phi = (m_1 - m_2) / (1 + m_1 m_2)$, where ϕ is the angle between the focal radius of either of the points of tangency and the line joining the focus and the external point, and m_1 and m_2 are the slopes of these two lines.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Designate the coördinates of the point of contact A by x_1, y_1 , and those of the point of contact B , x_2, y_2 , the equation of the ellipse being $a^2 y^2 + b^2 x^2 = a^2 b^2$; then the equation of tangents PA and PB will be, respectively, $a^2 y y_1 + b^2 x x_1 = a^2 b^2$, and $a^2 y y_2 + b^2 x x_2 = a^2 b^2$. By solving these as two simultaneous equations we find

$$x = \frac{a^2(y_2 - y_1)}{x_1 y_2 - x_2 y_1}, \quad y = -\frac{b^2(x_2 - x_1)}{x_1 y_2 - x_2 y_1},$$

which are the coördinates of the point P . Since the coördinates of focus F are $-ae$ and 0 , we find the slope of PF

$$= -\frac{\frac{b^2(x_2 - x_1)}{x_1 y_2 - x_2 y_1}}{\frac{a^2(y_2 - y_1)}{x_1 y_2 - x_2 y_1} + ae} = -\frac{b^2(x_2 - x_1)}{a[ay_2 - ay_1 + e(x_1 y_2 - x_2 y_1)]},$$

and the slope of $AF = \frac{y_1}{x_1 + ae}$.

$$\therefore \tan PFA = \left[-\frac{b^2(x_2 - x_1)}{a[ay_2 - ay_1 + e(x_1 y_2 - x_2 y_1)]} + \frac{y_1}{x_1 + ae} \right] \\ \div \left[1 - \frac{b^2 y_1 (x_2 - x_1)}{a(x_1 + ae)[ay_2 - ay_1 + e(x_1 y_2 - x_2 y_1)]} \right]$$

$$= -\frac{(a+ex_1)[a^2y_1y_1+b^2x_1x_2-a^2b^2]}{a^2(a+ex_1)[x_1y_2-x_2y_1-ae(y_1-y_2)]} = -\frac{a^2y_1y_2+b^2x_1x_2-a^2b^2}{a^2[x_1y_2-x_2y_1-ae(y_1-y_2)]}.$$

It is seen that this expression is symmetrical with reference to x_1 and x_2 , y_1 and y_2 with the exception of the sign, but considering that by finding $\tan PFB$ the slope of BF comes first, it is at once seen that $\tan PFB$ is the same as $\tan PFA$. The difficulty of this method lies in the complicated algebraic work, which is avoided by using polar coördinates.

Solution of 255 by Prof. William Hoover was received after the solution in last issue had gone to press. Also a solution of 256 was received from a contributor who failed to sign his name.

NOTE. Professor Matz sent in a solution of 254 in which he points out that the line $x-4a=0$ is both tangent and normal to the curve. But the solution is not general. Who can give a general solution?

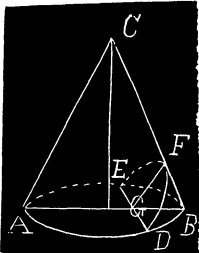
CALCULUS.

195. Proposed by CHRISTIAN HORNING, Heidelberg University, Tiffin, O.

Given a right cone of altitude h and radius r , to locate the plane parallel to its side which bisects the cone.

Solution by A. H. HOLMES, Brunswick, Maine, and J. SCHEFFER, Hagerstown, Md.

Let, in the right cone CAB , DEF represent a parabolic section. Put $BG = x$, $GE = y$, $FG = z$. The area of $DEF = \frac{4}{3}yz$; and consequently the volume of



$$BDEF = \frac{4}{3} \cdot \frac{h}{\sqrt{(r^2 + h^2)}} \int_0^x yz dx,$$

and since $y^2 = 2rx - x^2$, and $z = \frac{x}{2r} \sqrt{(r^2 + h^2)}$, we have for the volume, the integral

$$\frac{2}{3} \frac{h}{r} \int_0^x x dx \sqrt{(2rx - x^2)} = \frac{2}{3} \frac{h}{r} \left[\frac{1}{2} r^3 \cos^{-1} \frac{r-x}{r} - \frac{3r^2 + rx - 2x^2}{6} \sqrt{(2rx - x^2)} \right].$$

To determine x for the condition that this volume is to be half the cone, we have the transcendental equation

$$2r^3 \cos^{-1} \frac{r-x}{r} - \frac{2}{3} (3r^2 + rx - 2x^2) \sqrt{(2rx - x^2)} = r^3.$$

An approximate value of x is $x = 1.3r$.

Also solved by R. D. Carmichael.

196. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

The shortest tangent intercepted by the axes of the ellipse to which the tangent is drawn, equals the sum of the semi-axes of the ellipse.